國立交通大學

Transceiver Designs for Multicarrier Transmission

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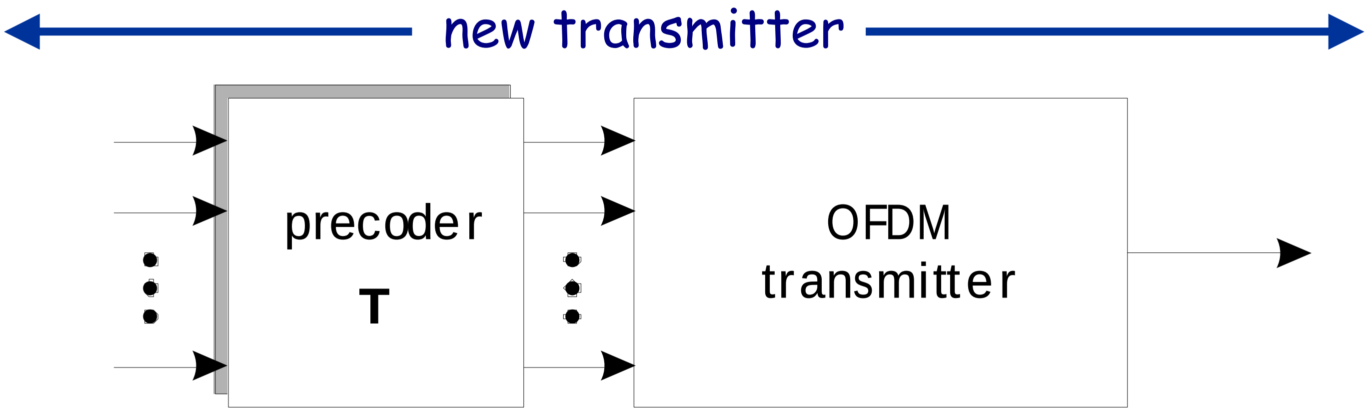
OFDM systems with precoders

transmitter of a general block transceiver by precoding:



(orthogonal frequency division multiplexing)

Aim: Design the transceiver (channel independent transmitter for minimum BER (bit error rate)

Earlier works

Block transceivers optimal in various senses have been investigated

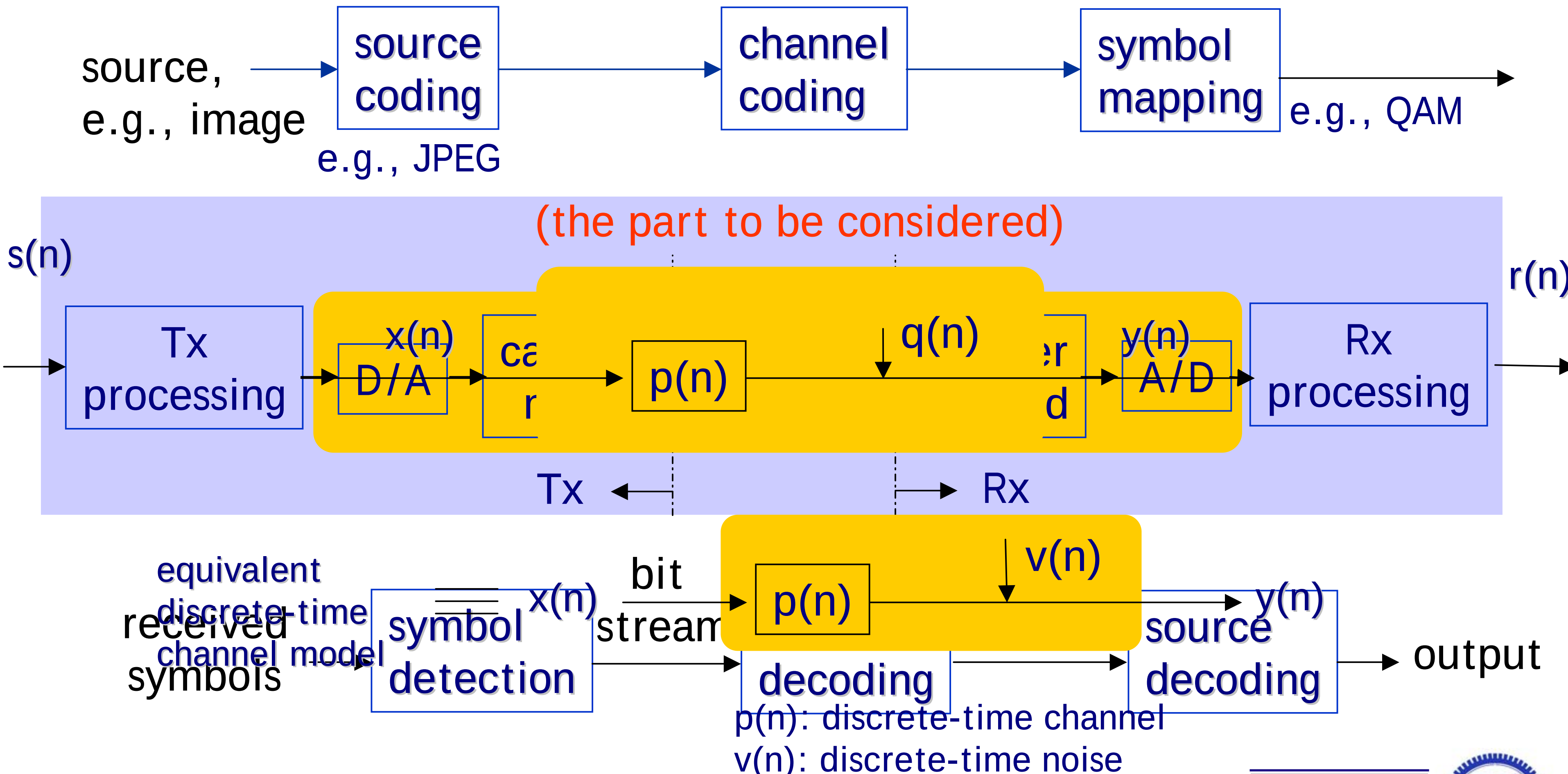
- minimum mean square error
  Scaglione, Giannakis, Barbarossa, 1999,
  Palomar, Cioffi, Lagunas, 2003
- maximum information rate
  Al-Dhahir and Cioffi, 1996, Scaglione, Giannakis, Barbarossa, 1999
- maximum bit rate,
  Yasotharan, 2006
- minimum transmission power,
  Lin, Phoong, 2001
- minimum bit error rate
  Ding, Davidson, Luo, Wong, 2003, Lin, Phoong, 2003

** In most earlier works, the transmitters are channel dependent.

Outline

- Introduction and preliminaries
  - channel model
  - guard interval, block transmission
- OFDM systems
- SC-CP systems
- Precoded OFDM systems
- Zero-forcing precoded OFDM systems
- MMSE precoded OFDM systems
- Examples and conclusion

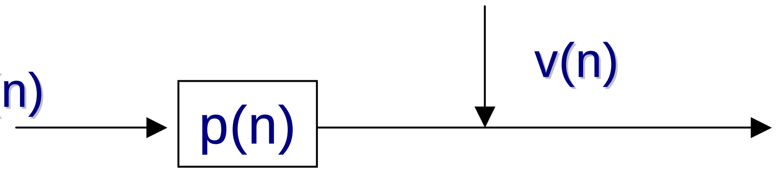
Modern Communication System



Ex. AWGN channel, QPSK

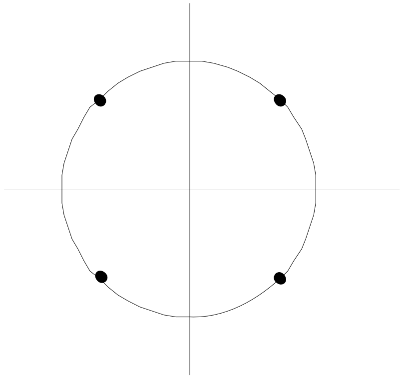
AWGN: P(z)=1
v(n): white Gaussian noise, variance N_0

-- no Tx processing, no Rx processing



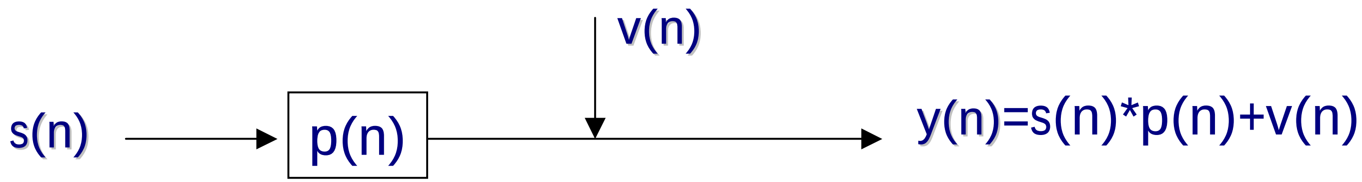
-- s(n): QPSK, variance Es

$$s(n) = \pm\sqrt{\frac{E_s}{2}} \pm j\sqrt{\frac{E_s}{2}}; \quad |s(n)| = E_s$$



BER (bit error rate) $P = Q\left(\sqrt{\gamma}\right); \quad \text{where} \quad \gamma = E_s/N_0$

ISI channel



suppose P(z) is FIR, order L: $P(z) = p(0) + p(1)z^{-1} + \cdots + p(L)z^{-L}$

$$y(n) = \sum_{k=0}^{L} p(k)s(n-k) + v(n)$$

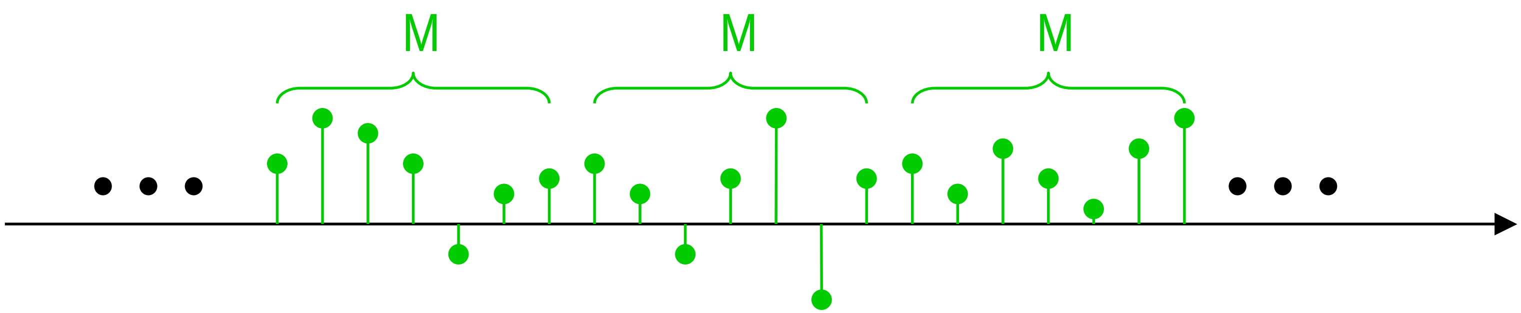
$$= p(0)s(n) + p(1)s(n-1) + \cdots + p(L)s(n-L) + v(n)$$

L ISI (inter-symbol interference) terms

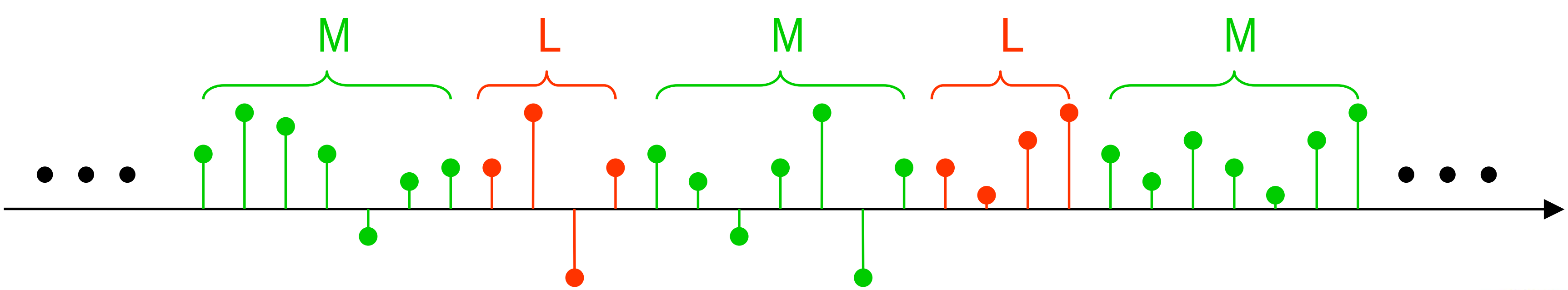
- before symbol-by-symbol detection, ISI terms need to be removed—channel equalization, which can be done using Tx/Rx processing

Using Guard Intervals for ISI Control

* partition into blocks of size M



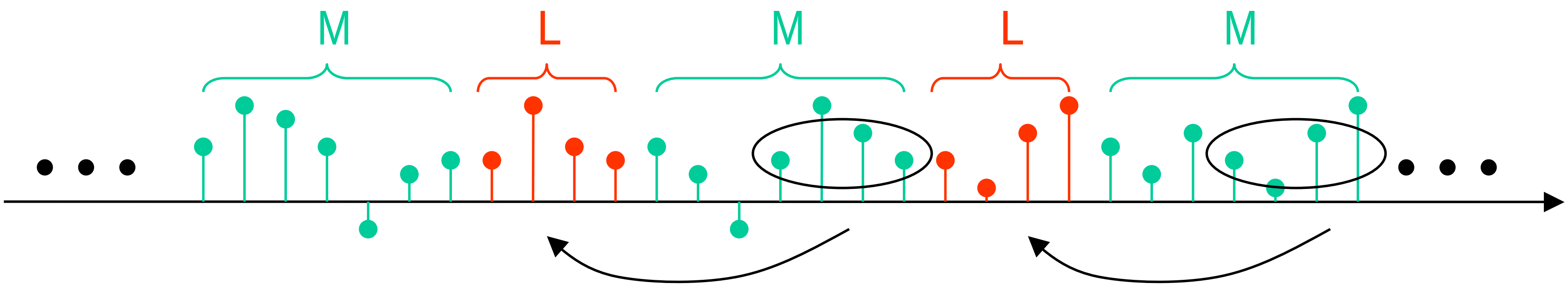
* insert guard interval of length L between every 2 blocks, samples in guard interval usually depends on the following block



A commonly used guard interval

★ **cyclic prefixing:**

the prefix is obtained by copying the last L samples from each block



IBI Free Property

- guard interval acts as a buffer between adjacent blocks
- no need of considering inter-block interference (IBI), i.e., IBI free
- only need to consider the transmission of a single block—one shot block transmission, though samples are transmitted consecutively
- ISI from only the same block

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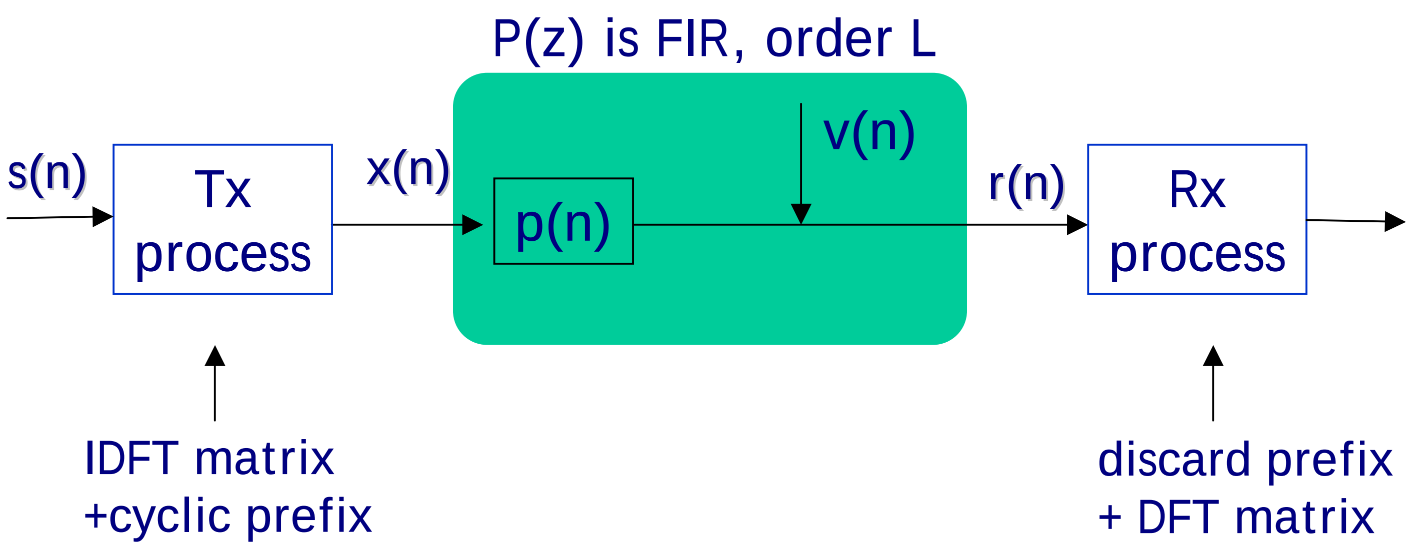
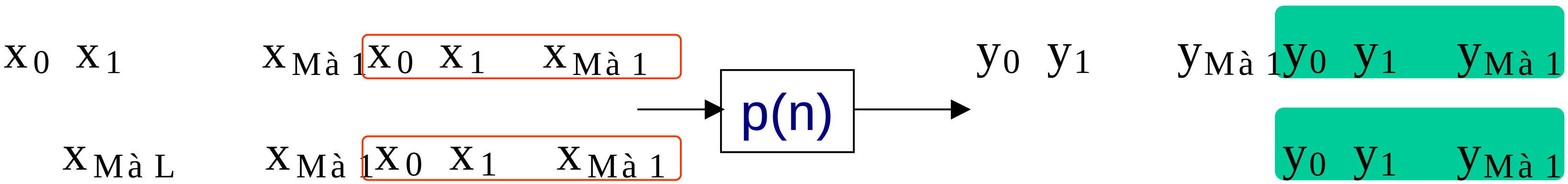
OFDM: DFT based Transceivers with Cyclic Prefix

- using IDFT matrix for Tx processing and DFT matrix for Rx processing

- overall system converted to M parallel AGN (additive Gaussian noise) subchannels

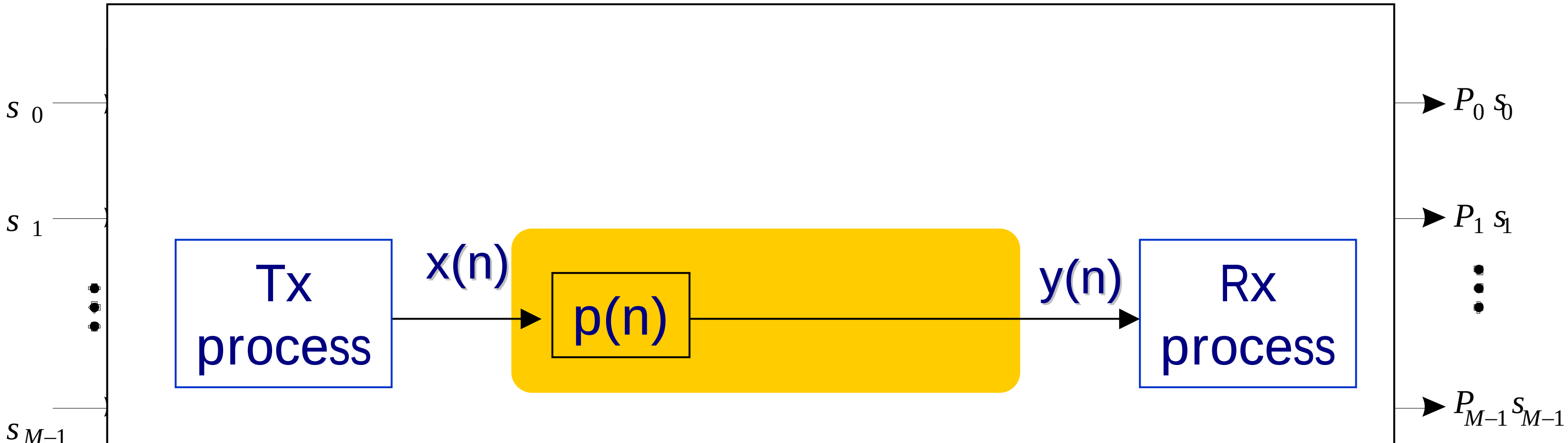


OFDM: DFT based Transceivers with Cyclic Prefix



when there is no noise

the same M points



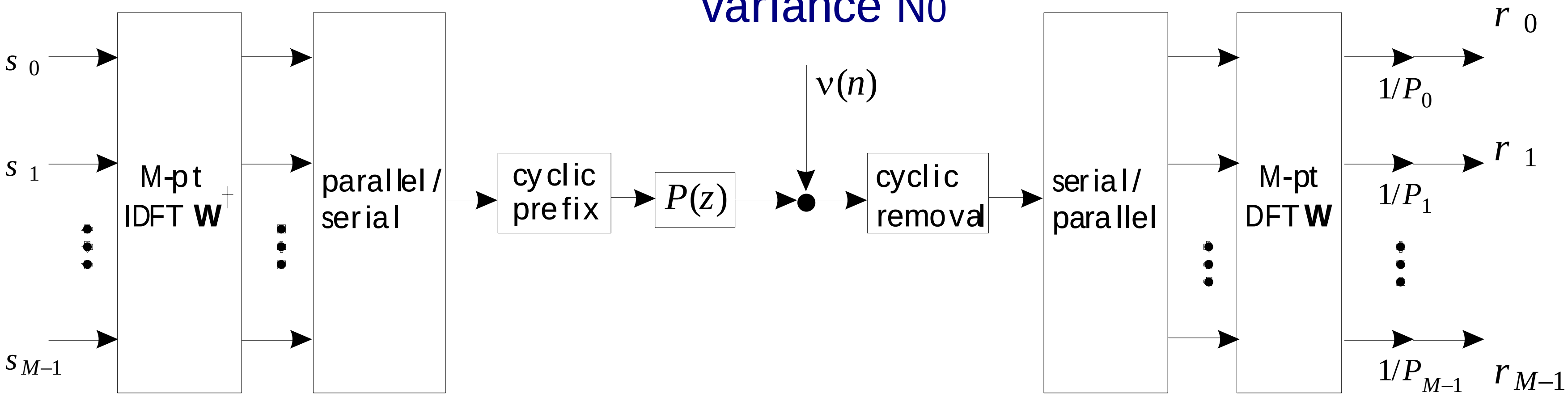
The outputs are scaled versions of the inputs when there is no noise.

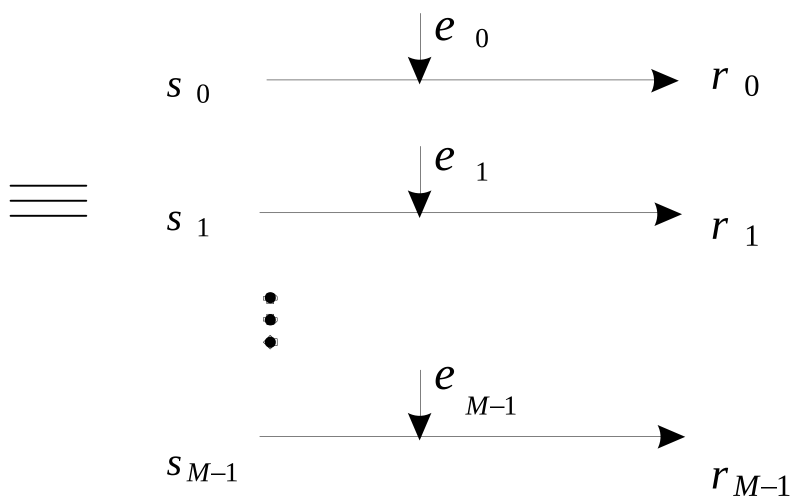
$s_k = X_k$

$Y_k = P_k X_k$

OFDM System

white Gaussian noise,
variance N_0





- ISI free
- $r_i = s_i$, in the absence of channel noise v(n)
- e_i: Gaussian noise
  variance $N_0{=}jP_ij^2$
  as DFT is unitary

BER of OFDM System

channel noise $\div(n)$: AWGN; zero mean; variance = N_0

input symbols s_k : QPSK symbol; $E[j s_k j^2] = E_s$

$$\text{SNR} \quad í = E_s = N_0$$

subchannel noise: $$û^2_{ofdm}(i) = \frac{N_0}{jP_ij^2}$$

average noise: $$E_{rr} = \frac{N_0}{M} \sum_{i=0}^{M-1} \frac{1}{jP_ij^2}$$

subchannel SNR: $$ì_{ofdm}(i) = í\, jP_ij^2$$

BER: $$P_{ofdm} = \frac{1}{M} \sum_{i=0}^{M-1} Q\, \text{à}\sqrt{í\, jP_ij^2}\,\text{á}$$

Advantages of OFDM Systems

- Channel diagonalization.

  ISI channels converted to parallel additive noise channels.

- Low complexity.

  IDFT at the transmitter and DFT at the receiver.

- Channel independent transmitter.

  No need of sending back channel profile to determine the transmitter. A property vital for wireless communications where the channel varies rapidly or for broadcast applications where there is on transmitter and many receivers.

- Low channel dependence receiver.

  Only a set of M scalars are channel dependent.

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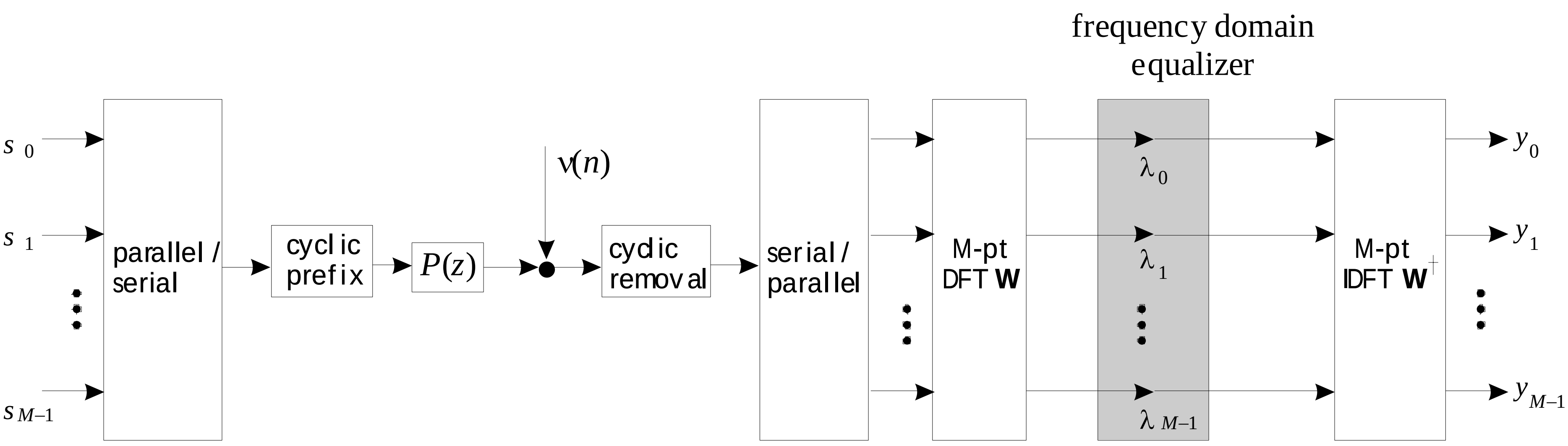
SC-CP (Cyclic-Prefixed Single Carrier) Systems

-- Also known as SC-FDE
(single carrier with frequency domain equalizer)



Connection of SC-CP to OFDM Systems

- can be obtained from OFDM by moving IDFT at transmitter to the end of receiver
- same overall complexity as OFDM, 2 DFT + FDE
- low PAPR (peak to average power ratio) as symbols are sent directly

  (In OFDM system, symbols are passed though an IDFT matrix before transmission. As a result, transmitted signals have Gaussian like distribution, and a large PAPR.

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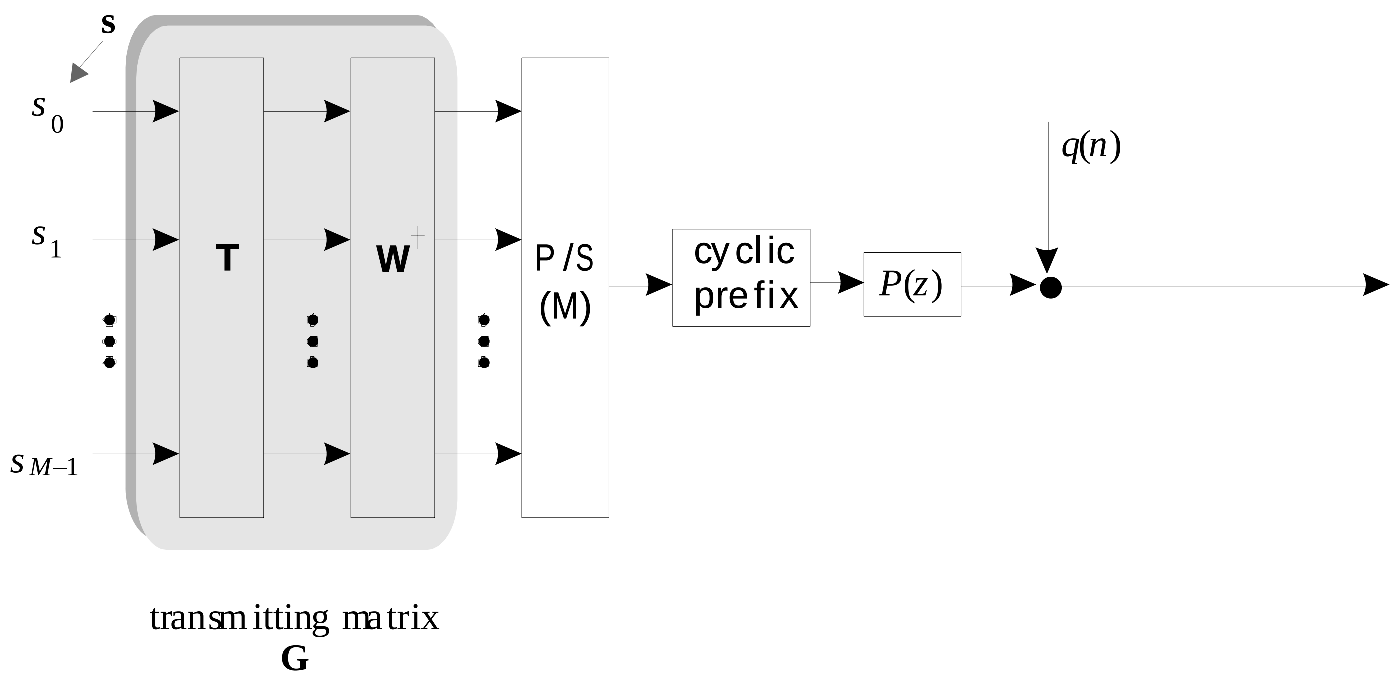
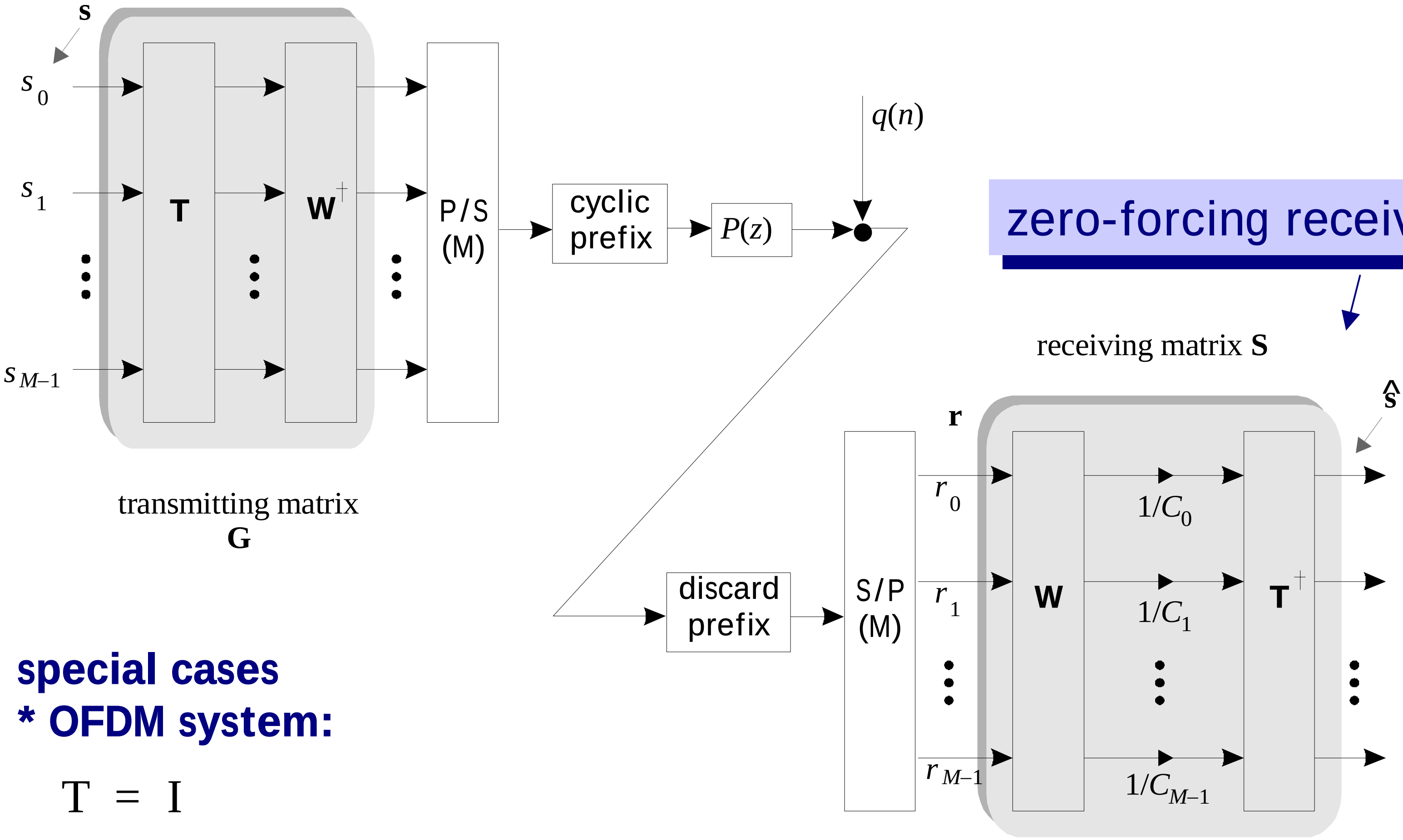
Problem of optimal precoders



Q: How to design unitaryT such that BER is minimized for QPSK?
Can we have a channel independent T?

* For multicarrier/block transmission system, MMSE does not necessarily minimize BER

Special Cases of precoder T



special cases

*** OFDM system:**

$$T = I$$

- **SC-CP system**

$$T = W \quad \text{(DFT matrix)}$$

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BER of Zero-forcing Precoded OFDM

(1) P_{ofdm} ô P_T ô $P_{scà\,cp}$; for í ô $í_0 = \min_i \frac{3}{jP_ij^2}$
(*)

(2) P_{ofdm} õ P_T õ $P_{scà\,cp}$; for í õ $í_1 = \max_i \frac{3}{jP_ij^2}$
(*)

* In each case, the 2nd inequality becomes an equality if and only if all subchannels have same SNR

- OFDM is the optimal solution for low SNR range 0
- SC-CP is the optimal solution for high SNR range 1
- OFDM is better for low SNR range and SC-CP is better for high SNR range.
- precoder OFDM has performance sandwiched between OFDM and SC-CP in either SNR range, optimal precoder is SNR dependent

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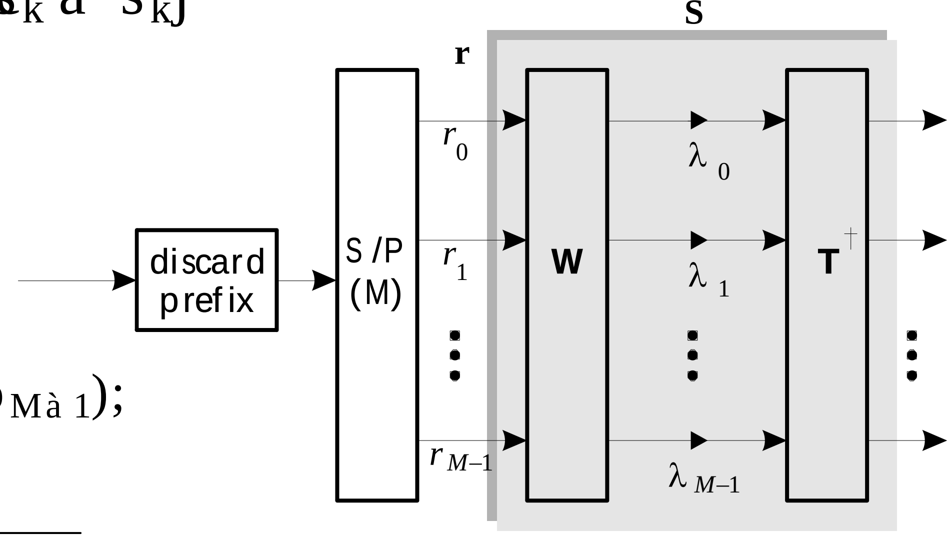
MMSE receivers

Lemma: The optimal receiving matrix S that minimize the average mean square error

$$E_{rr} = \frac{1}{M}\sum_{k=0}^{M-1} E\left[\,|\hat{s}_k - s_k|^2\right]$$

is given by



$$S = T^{\dagger}\Lambda W$$

$$\Lambda = \mathrm{diag}(\lambda_0 \ \lambda_1 \ \cdots \ \lambda_{M-1});$$

where $\lambda_i = \dfrac{\gamma P_i}{(1+\gamma|P_i|^2)}$

* same structure as a zero-forcing receiver

BER with MMSE receiver

* subchannel errors: interference + channel noise
* SINR (signal-to-interference-noise ratio):

$$\gamma(i) = \left(\sum_{k=0}^{M-1} \frac{|t_{k,i}|^2 \eta |P_k|^2}{1+\eta |P_k|^2}\right) \Big/ \left(\sum_{k=0}^{M-1} \frac{|t_{k,i}|^2}{1+\eta |P_k|^2}\right)$$

* BER:

Gaussian distribution, a good assumption for subchannel error [*] although subchannel errors is a mixture of interference and channel noise

$$P_{T;mmse} = \frac{1}{M}\sum_{i=0}^{M-1} Q(\sqrt{\gamma(i)})$$

[*] H. Vincent Poor, and Sergio Verdu, ``Probability of Error in MMSE Multiuser Detection", IEEE Trans. Information Theory, May 1997.

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BER of MMSE Precoded OFDM

$P_{sc\grave{a}\ cp;mmse} \quad P_{T;mmse} \quad P_{ofdm};$ for all SNR í :

* The first inequality becomes an equality if and only if subchannel SINRs β(i) are equalized.

- The SC-CP system has the smallest BER, and the conventional OFDM system is the worst solution
- The precoder achieves the smallest error rate, if and only if SINR β(i) are equalized.
- SINR's can be equalized by choosing a precoder that has the equal magnitude property.

$$|t_{m,n}| = \frac{1}{\sqrt{M}}, \quad 0 \leq m, n \leq M-1.$$

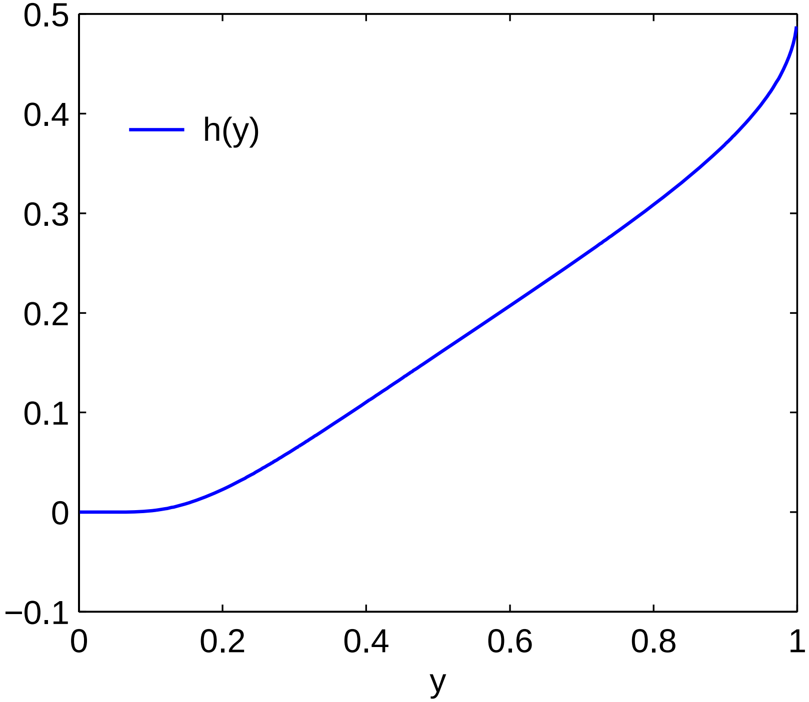
for example, Hadamard matrix and DFT matrix (channel independent)

BER of MMSE SC-CP, OFDM systems

Def: $h(y) \triangleq Q(\sqrt{y^{-1} - 1})$

$h(y) = Q(\sqrt{y^{-1} - 1})$ is convex

$h'(y) > 0; \quad h''(y) \geq 0; \quad 0 \leq y \leq 1$



BER in terms of h(.):

$$P_{sc-cp,mmse} = h\left(\frac{1}{M}\sum_{k=0}^{M-1} \lambda_k\right)$$

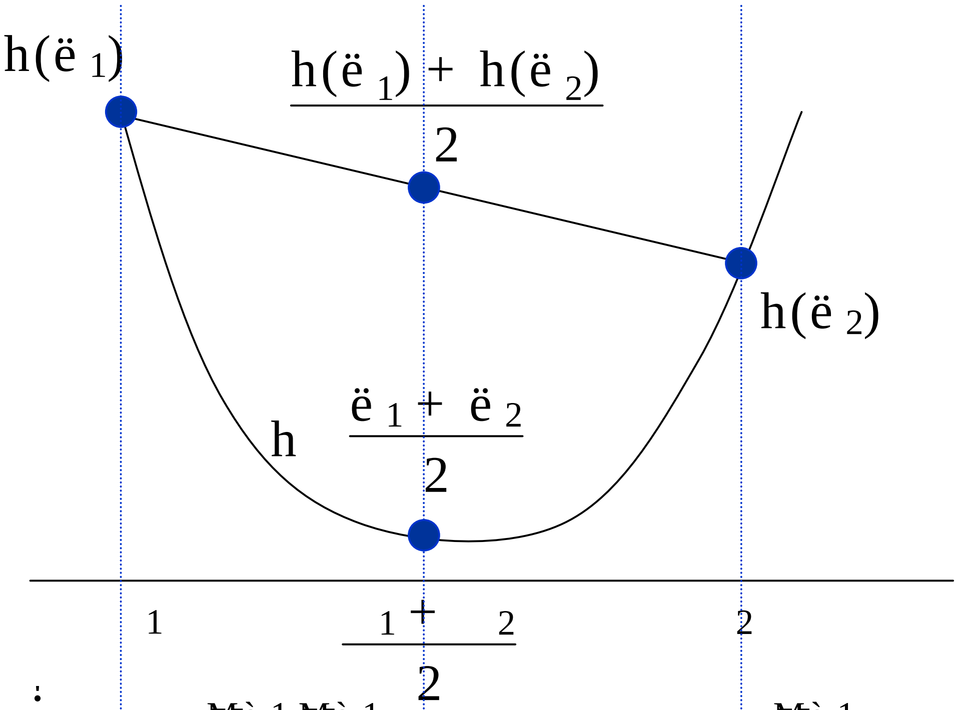
$$P_{T,mmse} = \frac{1}{M}\sum_{i=0}^{M-1} h\left(\sum_{k=0}^{M-1} |t_{k,i}|^2 \lambda_k\right)$$

$$P_{ofdm,mmse} = P_{ofdm} = \frac{1}{M}\sum_{k=0}^{M-1} h(\lambda_k)$$

where $\lambda_k = \dfrac{1}{1 + \gamma |P_k|^2}$

Proof:

using convexity of h(.)



$$\frac{h(ë_1) + h(ë_2)}{2} \; õ \; h \left(\frac{ë_1 + ë_2}{2} \right)$$

$$h\left(\frac{1}{M}\sum_{k=0}^{M à 1} ë_k\right) \; ô \; \frac{1}{M}\sum_{i=0}^{M à 1} h\left(\sum_{k=0}^{M à 1} jt_{k;i}j^2 ë_k\right) \; ô \; \frac{1}{M}\sum_{i=0}^{M à 1}\sum_{k=0}^{M à 1} jt_{k;i}j^2 h(ë_k) = \frac{1}{M}\sum_{k=0}^{M à 1} h(ë_k)$$

$$\sum_{i=0}^{M à 1} jt_{k;i}j^2 = 1$$

$P_{scà cp;mmse}$ $P_{T;mmse}$ P_{ofdm}

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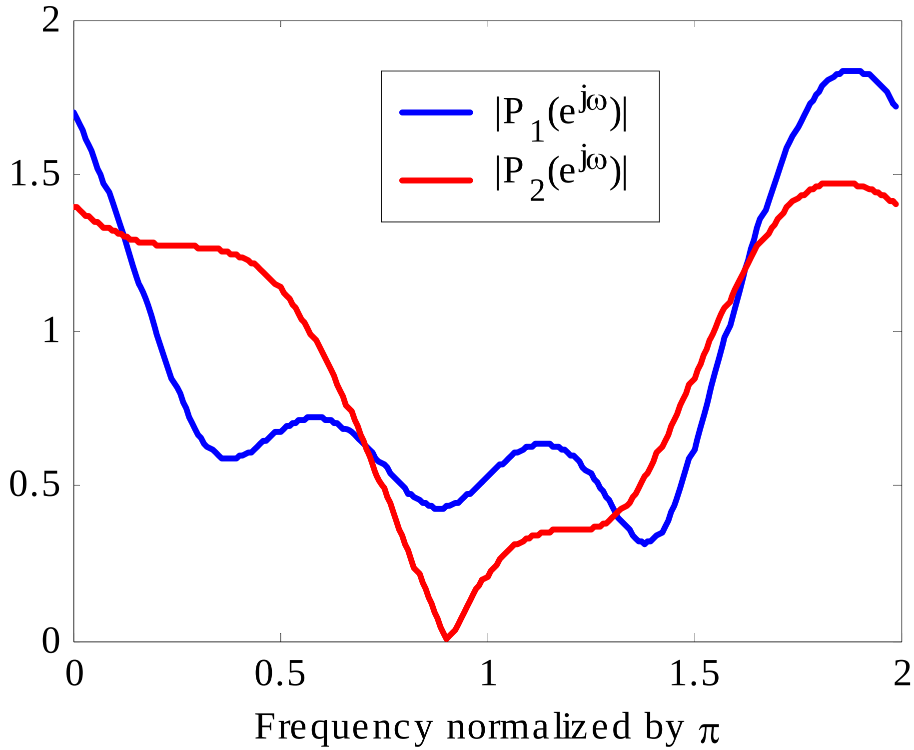
Example 1

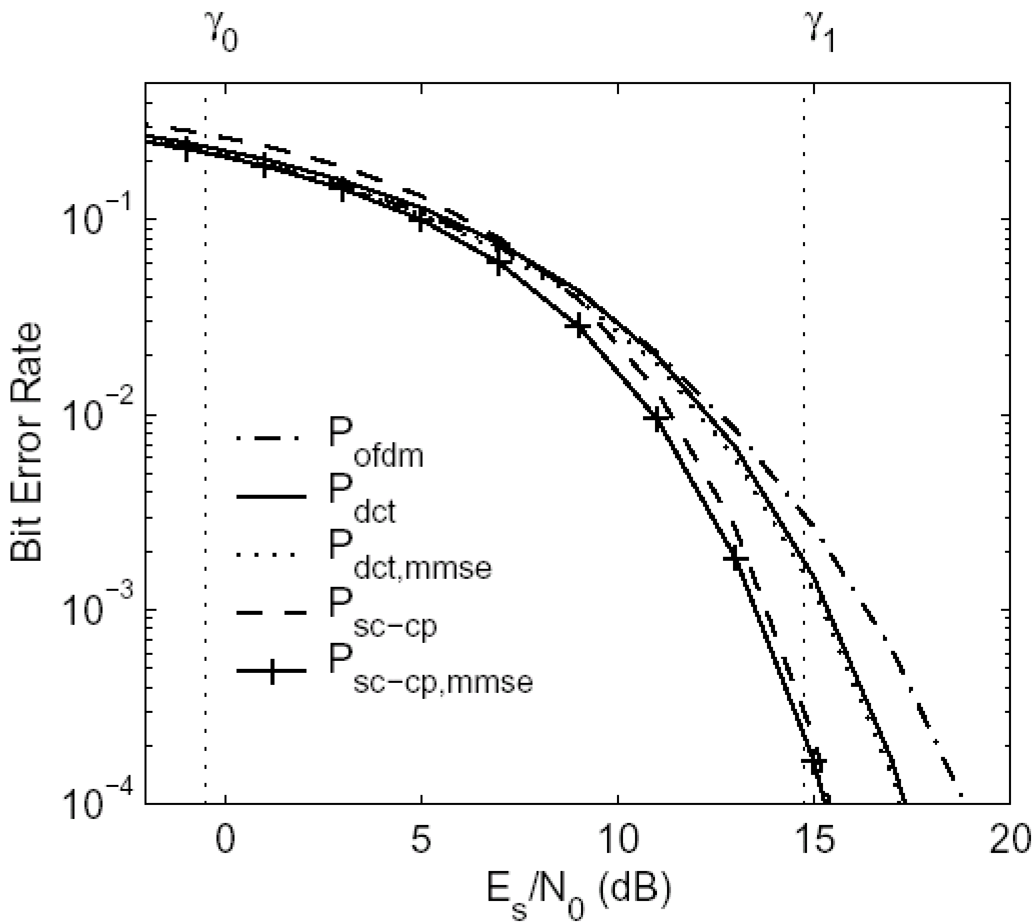
channel (4 coeffcients), M=64, L=3

p1(n): 0.3903 +j0.1049, 0.6050+j0.1422, 0.4402+j0.0368, 0.0714+j0.5002
p2(n): -0.3699-j0.5782, -0.4053-j0.5750, -0.0834-j0.0406, 0.1587-j0.0156



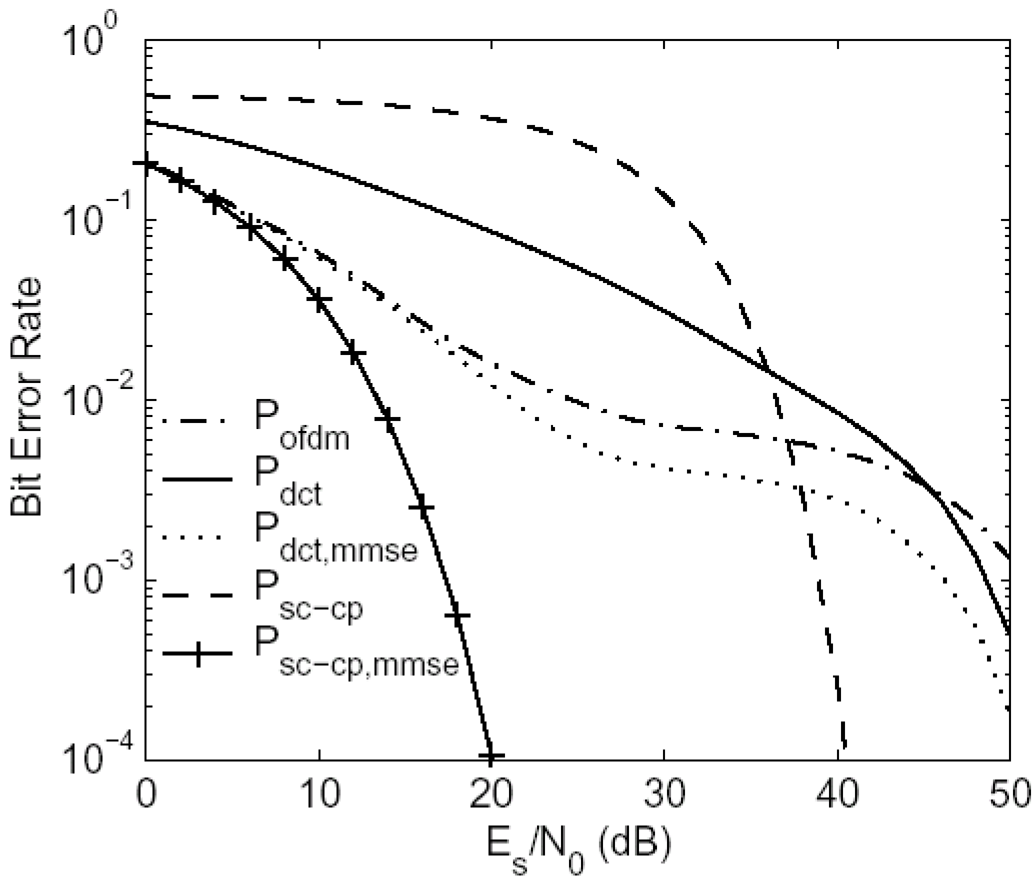
channel 2 has a zero around 0.9π

Example 1, channel 1



- MMSE SC-CP, DCT below OFDM for all SNR

Example 1, channel 2

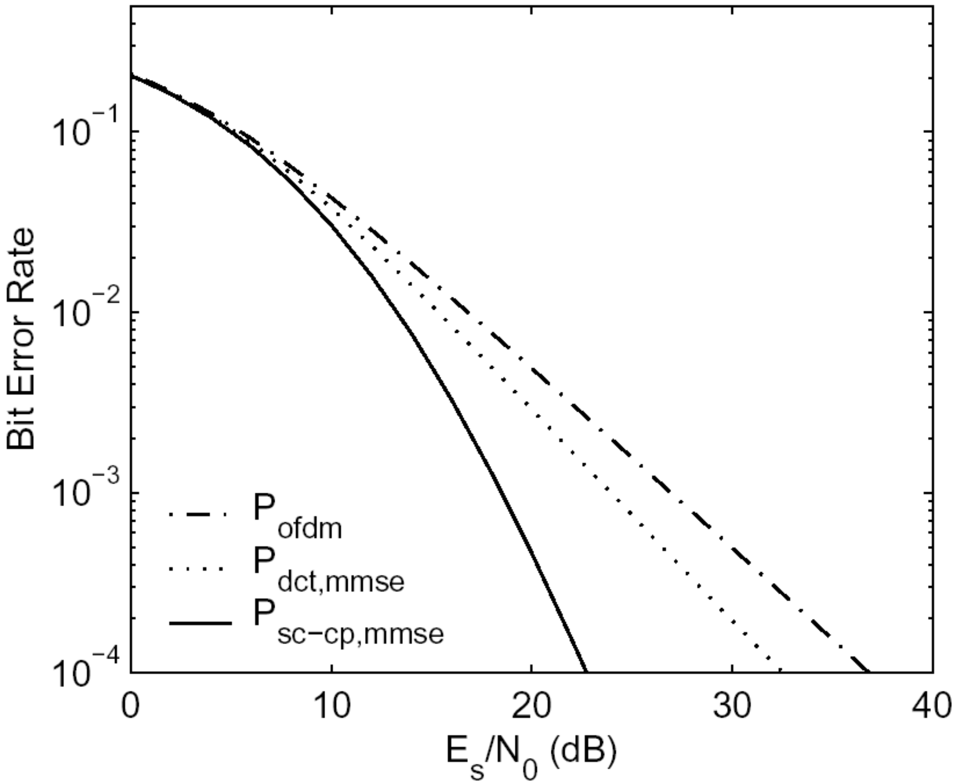


- for channel 2, the zero around 0.9π significantly degraded performances of SC-CP and OFDM systems, but not MMSE SC-CP

Example 2

multipath fading channel, 4 coefficients

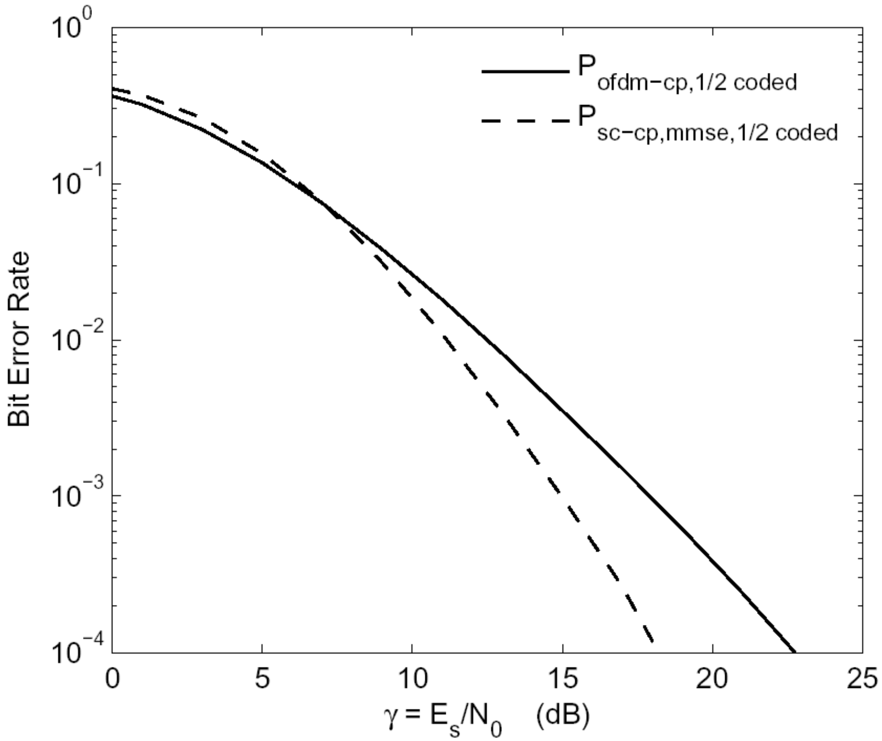
coefficients : independent complex Gaussian, zero mean, variances 8/15, 4/15, 2/15, 1/15

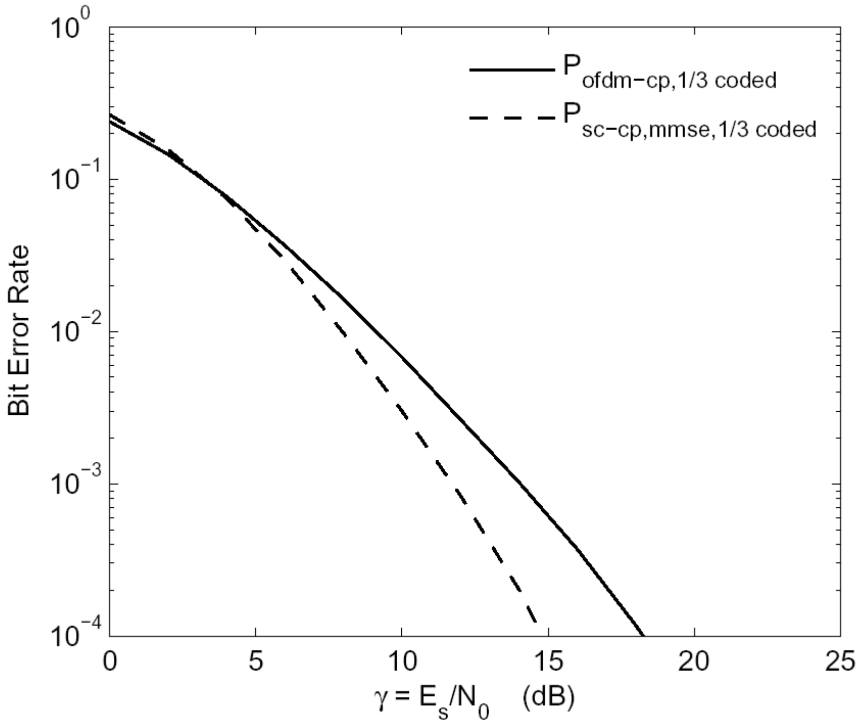


for BER=1.e-4,
the SNR gap between MMSE SC-CP and OFDM is around 15dB

Example 2

same multipath fading channel,
the input symbols are coded using convolutional codes

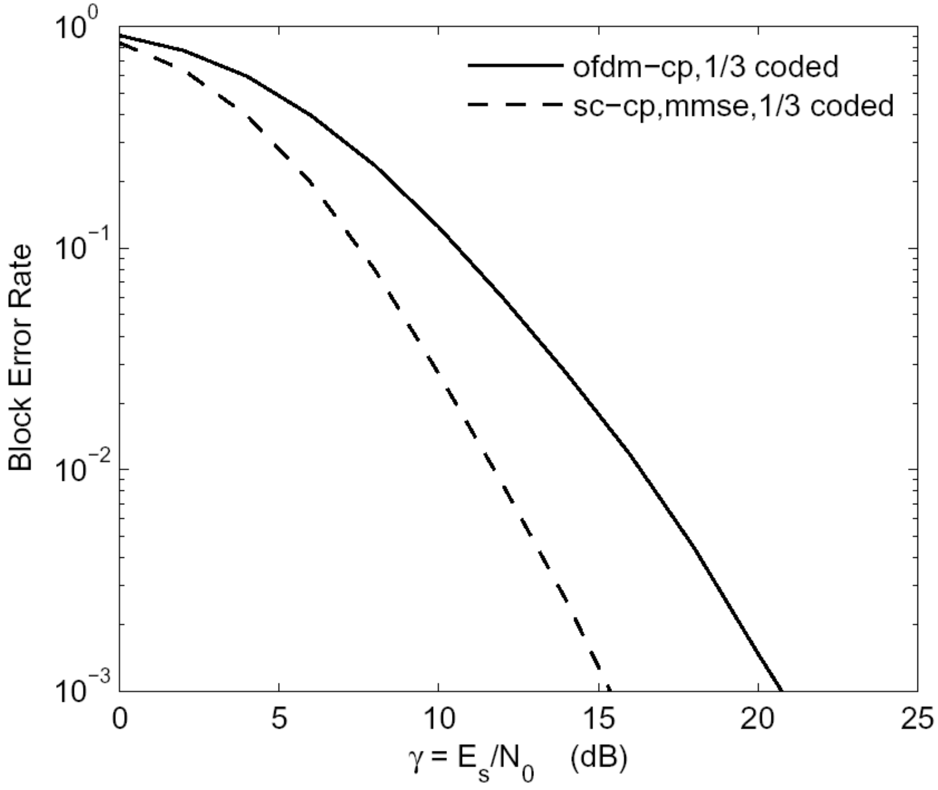




for BER=1.e-4,
the SNR gap narrows to 3dB for 1/3 coded case

Example 2, block error rate, coded symbols

block error rate, sometimes a more relevant measure
because an uncorrected bit error usually
means the whole block needs to be re-transmitted.



Concluding Remarks

For uncoded QPSK symbols,

- Optimal zero-forcing precoder depends on the SNR
- BER of an MMSE precoded OFDM systems between SC-CP and OFDM

For coded QPSK symbols:

- BER: whether OFDM or MMSE SC-CP is better depends on the coding rate and SNR.
- block error rate: for the same BER, MMSE SC-CP usually better as errors distributed more unevenly

Concluding Remarks

- Larger constellation QAM symbols:
  for both zero-forcing and MMSE cases
  the optimal precoder depends on the SNR γ,
  like the zero-forcing precoded system for
  QPSK symbols.

  For a practical BER range, the SC-CP MMSE
  system is better.